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Exponential form to rectangular form example

by M. Bourne IMPORTANT: In this respect ρ must be expressed in radians. We use a significant constant e = 2,718 281 8..., in this respect. We first met in part of Natural LogAriths (that basis e). The exponential form of a complex number is: r e^(\j\theta) (r is the absolute value of a complex number, as we were before in Polar Form; Answer We have r = 5 questions. We need to radiate ' ρ = 135^@' in radians. Remember: '1^text(o)=pi/180' So 135^text(o)=(135pi)/180' '=(3pi)/4' '~~2.36' radians, so that we can write '5(cos\ 1 35^text(o)+j\sin135^text(o))' '=5e^((3pij)/4)' ~~5e^(2.36j)' Example 2 Express -1 + 5j in an exponential form. Answer Complex number '-1 + 5j' We need to find ρ radians (see Trigonometric functions at any angle, if you need a reminder of reference angles) and r. 'alpha=tan^(-1)(y/x)' '=tan^(-1)(5/1)' '~~1.37text(radians) [This is 78.7^@when we worked in degrees.] Since our angle is in another sector, we need to implement: theta = pi - 1.37 ~~1.77' And r=sqrt(x^2+y^2)' '=sqrt((-1)^2 + (5)^2)' '= sqrt(26)' '~~5.10' So exponential form is 5.10e^(1.77j)' SUMMARY: Complex number forms These expressions have the same value. They are just different ways to express the same tricky number. A. Rectangular form x + yj b. Polar form r(cos ρ + j sin ρ) = r cis ρ = r∠ρ ρ may be in degrees or radians for the polar form. c. The exponential form of rejλ ρ must be an exponential form in radians. Exercises 1. Express in exponential form: '4.50(cos\ 282.3^@+ j\sin\282.3^@)' Reply '282.3^@ = 4.93' Radians Both '4.50(cos\ 282.3^@ + j\sin\282.3^@) ' = 4.50e^(4.93j)' 2. Express in exponential form: -1 - 5j Answer 3. Express in polar and rectangular form: 2.50e^(3.84j) Answer We can immediately write: 2.50e^(3.84j) = 2.50\ /_ \ 3.84 [polar form, ρ in radians] OR, if you want, because '3.84\ radians = 220^@', '2.50e^(3.84j) ' = 2.50(cos\220^@ + j\sin\220^@)' [polar form, ρ degrees] And with this result we can multiply the right side to give: 2.50(cos\ 220^@ + j\ sin\ 220^@)' = -1.92 -1.61j Summary Our complex number can be written in the following equivalent forms: 2.2.2.2.2.61j 50e^(3.3.20000) 84j)' [exponential form] '2.50\ /_ \ 3.84' '=2.50(cos\220^@ + j\sin\ 220^@) [PolarForm] '-1.92 -1.6 1 j' [rectangular form] Euler's Formula and Identity The next part is an interactive graph where you can explore a specific case of complex numbers in an exponential form: Euler Formula and Euler Identity Interactive Graph Show Mobile Message All Notes Hide All Notes Mobile Message You seem to have a device with narrow screen width (i.e. you're probably a mobile phone). Thanks to the nature of the mathematics of this site has the best views in landscape mode. If your device is not in landscape mode, many equations run off the side of the device (it should be possible to see them) and some menu items are cut off because the screen width is narrow. Most people are familiar with complex numbers in the form of \(z = a + bi\), but there are some alternative forms that are useful at times. In this regard, we look at both them and a couple of nice facts that arise from them. Geometric interpretation before we get into alternative forms we should first take a very short look at the natural geometric interpretation of complex numbers, as this takes us to our first alternative form. Consider the complex number \(z = a + bi\). We can think of this complex number as either a point \(\left({a,b} \right)\) in the standard Descartes coordinate system or as a vector that begins at origin and ends at \(\left({a,b} \right)\). An example of this is shown in the figure below. In this interpretation, we call the axis \(x\) - the actual axis and axis \(y\) - axis. We often call a complex plane when it came to this interpretation. Note also that we can now get a geometric interpretation of modulus. In the image above, we can see that \(\left| z \right| = \sqrt {{a^2} + {b^2}} \) is nothing more than the length of the vector that we use to represent a complex number \(z = a + bi\). This interpretation also tells us that inequality \(\left| {{z_1}} \better| < \left| {{z_2}} \right|\) means that \({z_1}\) is closer to origin (on a complex plane) than \({z_2}\) is. Polar Form Let's now take a look at the first alternative form of the complex number. If we think of a zero-different complex number \(z = a + bi\) as a point \(\left({a,b} \right)\) in the \(xy\)-plane we also know, that we can represent this point with polar coordinates \(\left({r,\theta} \right)\), where \(r\) is the point distance from the source and \(\the\\) is the angle, in radians, from the positive axis \(x\) to the beam that connects the origin to the point. When working with complex numbers, we assume that \(r\) is positive and that \(\theta \) can have any possible (both positive and negative) angle that ends on the ray. Note that this means that \(\theta\) is literally an endless number of options. We exclude \(z= 0\) because \(\theta \) is not defined as a point (0.0). Therefore, we only take into account the polar form of unset complex numbers. We have the following conversion formulas to convert polar coordinates to \(\left({r,\theta} \right)\) to point cartesia coordinates, \(\left({a,b} \right)\). \[a = r\cos \theta \hspace{0.75in} b = r\sin \theta \] When we replace them \(z = a + bi\) and factor \(r\) out we reach the polar form complex number, \begin{ equation}z = r\left({\cos\theta + i\sin \theta} \right) \label{eq:eq1}\end{equation} Also note that we also have the following formula for polar coordinates associated with \(r\) to \(a\) and \(b\). \[r = \sqrt {{a^2} + {b^2}} \] but party is nothing more than the definition modulus and we see that \begin{equation}r = \left| z \better|\label{eq:eq2} \end{equation} So sometimes the polar form is written as \begin{equation}z = \left| z \right|\left({\cos \theta + i\sin \theta} \right)\label{eq:eq3}\end{equation} Corner \(\theta \) is called \(z\) and is indicated by \[\theta = \arg z\] Argument \(z\) may be an infinite possible value that can be found by solving \begin{equation}\tan \theta = \frac{b}\label{eq:eq4}\end{equation} and making sure \(\theta \) is in the correct sector. Note also that the two values in the argument differ from each other with an integer multiple count of \(2\pi \). This makes sense when you consider the following: For a given complex number \(z\), select a possible value for the argument, say \(\theta \). If you now increase the value \(\theta \), which really increases the angle that the point does with the positive \(x\) axis, you will rotate the origin of the point counterclockwise. Since it takes \(2\pi \) radians to make one complete revolution then return to your original starting point when you reach \(\theta + 2\pi \) and so there is a new value argument. See diagram below. If you continue to increase the angle, it will be back at the starting point when you reach \(\theta + 4\pi\), which is the new value of the argument again. Continuing in this way we can see that every time we reach the new value argument we will simply add the number of \(2\pi \) onto the original value argument. Also, when you start with \(\theta \) and reduce the angle then rotate the point around the origin clockwise way and returns to your original starting point when you reach \(\theta - 2\pi \). Continuing this way, and we see once again that each new value argument is found subtracting the multiple \(2\pi\) from the original value of the argument. So, we see that if \({\theta _1}\) and \({\theta _2}\) have two values \(\shy z\) then some integer \(k\) we have, \begin{equation}{\theta _1} - {\theta _2} = 2\pi k\label{eq:eq5}\end{equation} Note that we have also shown that \(z=r\left({\cos\theta + i\sin \theta }right) \)\) is a parametric representation of the circle radius \(r\) focused origin and that it is following the circle counterclockwise , when the angle increases \(\theta \) \(\theta + 2\pi \). The basic value of the argument (also called the main argument) is the unique value of the argument, located in the range of \(-\pi < \arg z \le \pi \) and is represented by \({mathop{\rm Arg}olimits} z\). Note that inequality between each end says that the argument for negative actual number is \({mathop{\rm Arg}olimits} z = \pi \). Recalling that we noted above: the two values of the argument differ from each other by a multiple of \(2\pi\), which leads us to the following fact: \begin{equation}\arg z = {\mathop{\rm Arg}olimits} z + 2\pi n \hspace{0.25in} n = 0, \pm 1, \pm 2, \ldots\label{eq:eq:6}\end{equation} We should probably make a few quick numeric examples before moving forward to look at another alternative form of the complex number. Example 1 Write down the polar form of each of the following complex numbers. \(z = - 1 + i\,\sqrt 3 \) \(z = - 9 \) \(z = 12\,i\) Show all solutions Hide all solutions Show solution Let's first get \(r\). \[r = \left| z \right| = \sqrt {1 + 3} = 2\] Now we find the argument \(z\). It can be any angle that satisfies \(\eqref{eq:eq4}\), but usually the easiest way to find a core value is that we can find it. The main value of the argument is between \(\ pi < \theta \le \pi \), satisfied, \[\tan \theta = \frac{{\sqrt 3 }}{{ - 1}} \hspace{0.25in} \Rightarrow \hspace{0.25in} \theta = {\tan ^{ - 1}}\left({ - \sqrt 3} \right)\] and is in another sector because it is a complex location on a complex plane. If you use a calculator to find this reverse tangent value, make sure that you understand that your calculator returns only values in the range \(\frac{\pi}{2} < \theta < \frac{\pi }{2}\), and therefore you may get the wrong value. Remember that if the calculator returns \({\theta _1}\), the second value corresponding to the equation is \({\theta _2} = {\theta _1} + \pi \). So, if you use a calculator be careful. You have to calculate this way and determine what falls in the right quadrant to match our complicated number, because only one of them is in the right sector. For us, these two values are: \[{\theta _1} = - \frac{\pi }{3} \hspace{0.25in} {\theta _2} = - \frac{\pi }{3} + \pi = \frac{{2\pi }}{3}\] The first is four meters and the second is in two sectors, and so is the one we are looking for. Therefore, the main value of the claim is \[{\mathop{\rm Arg}olimits} \,z = \frac{{{{2\pi}}{3}\] and all possible values of the argument are then \[\arg z = \frac{{2\pi}}{3} + 2\pi n \hspace{0.25in} n = 0, \pm 1, \pm 2, \ldots \] Now, we are actually doing what we originally did. Here is the polar form \(z = - 1 + i\,\sqrt 3 \). \[z = 2\left({\cos \left({\frac{{{2\pi}}{3}} \right) + i\sin \left({\frac{2\pi }}{3}\right)\] Now, for the sake of completeness, we should recognise that there are more equal polar forms for this complex number. To get other forms, we simply need to calculate the different value of the argument by selecting \(n\). Here are some other possible polar forms. \begin{ align*}z & = 2\left({\cos \left({\frac{{{{8\pi}} {3}} \right) + i\sin \left({\frac{8\pi }}{3}\right)} & \hspace{0.25in} & n = 1\\z & = 2\left \left({ - \frac{{16\pi }}{3}} \right) + i\sin \left({ - \frac{{16\pi }}{3}} \right)} & \hspace{0 .25in} & n = - 3\end{align*} show a solution In this case, we have already indicated that the core value of the negative actual number is \(\pi \), so we do not have to calculate it. For the sake of completeness, here is all possible values argument for any negative number. \[\arg z = \pi + 2\pi n = \pi \left({1 + 2n} \right) \hspace{0.25in} n = 0, \pm 1, \pm 2, \ldots \] Now, \(r\) is, \[r = \left| z \right| = \sqrt {81 + 0} = 9\] The polar form (using the core value) is \[z = 9\left({\cos \left(\pi \right) + i\sin \left(\pi \right)} \right)\] Note that if we would be a positive actual number, would be the core value \ {{{{\mathop\\\